PMU Measurement Technology

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April 2014
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Executive Summary

Phasor measurements are without doubt extraordinarily valuable and informative. The number of phasor measurement units in service continues to increase, to the great benefit of the power system. And yet the measurement method used is deeply flawed.

This report uses an epistemological approach to explain why the measurement of synchrophasors, as it is presently implemented, is unjustified.

Fundamentally, the problem is that the definition for the quantities being measured is incorrect. We expect the PMU to furnish values for the three parameters that define a phasor (its amplitude, frequency and phase), when in fact the signals we furnish to the PMU are not phasors. It follows that the best we can expect is an approximation. It also follows that we cannot know the quality of that approximation. As an example of what this means, consider that the PMU is required to furnish a value for the rate of change of frequency, and yet by the definition of a phasor, that rate of change is zero.

The report goes on to present an alternative method of describing the input signals to the measuring system. Instead of imposing the requirement that the signal be described by a phasor, the assumption of stationarity is relaxed, and the value of each of the three parameters is permitted to change during the measurement. The result of changing the definition is that a new measurement technique is possible.

As a byproduct of the new definition for the input signal, new (and concrete) definitions emerge for the terms “apparent frequency” and “instantaneous frequency.”

Preliminary tests of the new measurement method are given. They seem very promising.
Acknowledgements

The support of Phil Overholt at the US Department of Energy is gratefully acknowledged. Phil has played a major role in the advancing field of phasor measurements almost from the outset, and his understanding and encouragement have been constant and invaluable.

This report on work done on a new definition for the quantity being measured by a phasor measurement unit was the outcome of a considerable amount of out-of-the-box thinking, and a large number of discussions over an extended period with experts in the field. While acknowledging the contributions of all the members of the IEEE Power System Relaying Committee working group C37.11, the authors gratefully acknowledge in particular the help and interest of, and the many conversations with, Ken Martin of EPG, chair of the PMU standard working group, and Jay Murphy of Macodyne. The many discussions we had contributed significantly to the notions described here. The clear explanations given by Veselin Szendik of SEL have been of great benefit. Jerry Stenbakken of NIST is to be recognized for his starting one of us (Kirkham) down the path of pondering the “digital revolution in measurements,” a phrase that he may have originated, and that we have gratefully borrowed.
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PMU Measurement Technology

About this report

A few years ago, the first author of this report began to represent PNNL on the working group of the IEEE Power Engineering Society responsible for the development of a new standard for phasor measurement units, being written under the auspices of the Power System Relaying Committee. As a long-time member of the IEEE PES Instrumentation and Measurements Committee, he was somewhat familiar with the devices and with the standards-making process.

This was the first time that anyone from the Instrumentation and Measurement Committee had become deeply involved in the PMU effort. It soon became evident to him that there was a problem. The PMU was supposed to measure frequency and rate of change of frequency (ROCOF). And yet, as Carlson¹ put it long ago:

> These designations [frequency modulation and phase modulation] suggest time varying frequency or phase, concepts that require special interpretation. This is particularly true of time-varying frequency since frequency implies periodicity and time-varying periodicity is meaningless.

The problem that became evident was that the quantity being measured by the PMU was not well defined. The quantity apparently being measured by a PMU is a phasor, and yet by definition the rate of change of frequency of a phasor is zero. One might well ask what it is that is being measured, then.

Lack of a good definition for frequency and ROCOF implied that the result of the measurement by one PMU could be internally consistent and yet different from the result of the measurement of the same signal by a different PMU. Differences were indeed being observed, both during operation and during calibration. While such differences may have more than one cause, it did seem that a failure to base the measurement on a good definition was a strong possibility for the explanation.

The topic took several years to resolve. It is that resolution that forms the subject of this report. It may be that once the various steps along the way have been explained and discussed, the reader will conclude that the solution was obvious all along. Our only response to that is to say that many things do seem simple once they have been explained. The fact remains that the solution developed and reported here was not obvious to anyone else involved in the work during this time.

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¹ A. Bruce Carlson “Communication Systems” McGraw-Hill, 1968, New York. The quotation is from pages 222-223. The italics are in the original.
1.0 Introduction

The technology of power system measurement systems has evolved over the decades that power systems have been in existence. Early generators were made with the power instrumentation and controls and the voltage instrumentation and controls in separate rooms, because the two functions were viewed as essentially different. Of course, it was not long before the separate aspects of generator control were recognized as part of a system control, and they were united by a communication of information to one central place. That process has continued, with system controls becoming more and more integrated and more and more dependent on communications. A modern control center can access measurements from across the entire system. In the next section, we will briefly review some highlights of the evolution.

1.1 Power System Measurements Reviewed

At first, power system information was used only locally, but as the need for a central location of control was recognized, the data acquisition systems known as SCADA (for supervisory control and data acquisition) came into being. SCADA routinely gathered data every few seconds (typically a scan was made every four seconds) and reported the data to the control room.

Phasor measurements are similar in principle: the results of remotely-made measurements are centrally reported. However, the amount of information from one PMU so greatly exceeds the capability of SCADA that new communications schemes and data storage have to be used. Whereas a data-point in SCADA might be a measurement corresponding to the result of an eight-bit A/D conversion, and it would be reported every four seconds, a data point from a PMU would include a packet of data (called a frame) occupying several kilobytes. Frames of data are reported at standardized rates that could be as high as the power frequency itself.

Historically, as the voltage levels in power systems rose, instrument transformers became necessary to isolate the information system from the power system. For reasons of safety, practically all measurements in a power system are today made through instrumentation transformers. These devices are therefore part of the measurement system for SCADA and for PMU measurements.

1.1.1 Instrumentation Transformers

Instrument transformers have advantages in addition to isolating the measurement equipment from the high voltages in the power system. They also allow the range of the measurement equipment to be standardized. Thus, for most systems in the US, a primary voltage at its nominal value will produce a 120-volt signal at the secondary of a potential transformer. That holds for system voltages from distribution through transmission. Similarly, a current transformer furnishes a secondary signal of 5 Amps for a primary current at its nominal value.

---

2 The frame includes such details as the time of the measurement (derived from GPS), quality codes describing the adequacy of the time signal received from GPS, and a polynomial check word, in addition to data describing the results of the measurements made of voltages and currents, and their phase angles. For complete details, see IEEE C37-118.2™-2011, The IEEE Standard for Synchrophasor Data Transfer for Power Systems.
These values of voltage and current are high by comparison with the levels actually needed to make measurements using modern equipment, but for many years they brought with them a further advantage: they could provide the power needed to operate electromechanical relays. And because of that, they added (quite inadvertently) a kind of filtering effect that gave the electromechanical relay some immunity to the noise in the power system.

### 1.1.2 Noise in Power Systems

The noise immunity conferred by the combination of electromechanical relays and high power input signals came about principally by the fact that both the instrument transformer and the relay embodied low-pass filtering.

That filtering went almost unnoticed until transistorized relaying equipment was introduced into power systems in the 1960s. The noise in the signal was sometimes sufficient to destroy the transistors, and hence render the relay inoperative. The fact that destructive levels of noise were getting through the instrumentation transformer and to the relay meant that much of the low-pass filtering was taking place in the relay. The discovery of the noise problem did have two results of interest. First, it brought about an interest in the properties of the instrumentation transformers. The standards for these transformers specified only their behavior at power frequency, and they were generally tested at only that frequency. Second, it made engineers more aware of the need to develop what the mission assurance community calls “environmental requirements” for equipment entering service in the power system.3

We relate this topic to point out three things. First, we note that this story is an example of the introduction of new technology bringing to light unknown (and hence unexpected) aspects of the power system. Second, we remind the reader that the noise problem has not disappeared. Power systems are still noisy environments. We shall see later that both these aspects of the introduction of new technology were repeated when PMU technology was introduced. Third, we point out that the need for environmental requirements has not yet pervaded the power industry. PMUs, the subject of this report, are still not obligated to meet any particular environmental requirements, unless that obligation comes from a customer.

### 1.2 Phase Measurements in Power Systems

It is an odd fact that whereas the angle between the alternating voltages in various parts of the power system controls the power flow, angle has not been a parameter that was routinely measured. It is not a simple

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3 An environmental requirements document is generated for many aspects of operation in the military and in space, for example. The goal is to increase the likelihood of mission success. Those portions of a space system that must operate on the ground before and during launch are subject to requirements that are associated with operation in the thermal, dynamic or electromagnetic aspects of that regime. Those portions of the vehicle that must operate during entry, descent and landing are subject to requirements on dealing with descent winds and high pressure-change rates, for example. On Earth, depending on location, power system environmental requirements may include seismic withstand capability. That the system meets the environmental requirements is verified by environmental testing. However, for the most part, power systems have not systematized the use of environmental requirements as part of technology development. System specifications and acceptance tests rarely spell out environmental test procedures.
parameter to measure. The earliest reported method involved bringing a replica of the voltages at the two ends of a system to a single point, where they were compared. That was done in England in 1956 [1] to examine the system impact of faults on a line of about 150 miles (250 km) connecting a generators to the grid. The 50-Hz voltages of interest were modulated onto a carrier and transmitted to the laboratory via leased phone-quality cables. While the work was successful in many ways, it did not lead to any replications. It was simply too expensive to install all the cabling.

If one wanted to do that sort of measurement to look at the voltages between (say) southern California and the Pacific North-West, a total of at least 1500 km of communication links would be required to bring the signals to a single location for measurement. Extending this technique to measure all the possible angles between all the possible locations of interest in the power system would require an impossible amount of cabling.

Those early tests were done at a time that the atomic clock was still under development. The first successful clock was completed in June 1955 [2], but it was a somewhat unbelievable twelve years (i.e., 1967) before the full potential of the technology was accepted and the rotation of the planet measured in terms of the new standard second rather than the second being defined in terms of the rotation of the planet!

Between those years, Sputnik 1 was launched (October 4, 1957). Mark Adamiak, one of the creators of the first phasor measurement unit, has pointed out that all the elements necessary for the creation of GPS were present when Sputnik 1 started beeping. And GPS enabled PMU technology.

The availability of precise time, such as that obtainable from GPS, changes the problem of measuring the angle. While “angle” is still a parameter that requires a measurement of one signal with respect to another, knowing the time allows one of the signals to be a hypothetical one, constructed from the time signal just to be a reference. With the advent of GPS, this capability exists all over the world. The reference signal is at the nominal power frequency (50 or 60 Hz) and is defined as a cosine wave that has a positive maximum on the seconds-tick of UTC. With that definition, a reference wave can be constructed anywhere. The angle between two real locations can be calculated simply by subtracting one of the values of the angle to the reference from the other. Because the time is precisely known, the references are identical, and cancel.

To achieve the accuracy needed for the power system controls, the time signal accuracy must be a few tens of microseconds, easily obtained from GPS. In fact, the first Standard for the PMU (IEEE Std 1344 - 1995) chose an accuracy of 1 microsecond simply because GPS was capable of providing that level of timing accuracy.

1.3 State of the Art, 2014

Phasor measurement units are nowadays available for purchase from many manufacturers, and there are even open-source versions of the software. The technology of phasor measurement has evolved over the last few years, and the number of PMUs in use has increased considerably. There are now over a thousand installed in the US alone.

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4 While the delays involved in the transmission were not known, it was (reasonably) assumed that they were constant during the tests. Since it was the change in angle that was of interest, any constant offset could be discounted.
The installation of these measuring systems has exposed some surprises in the power system. For the most part, what has been revealed has been the result of the ability to “see” information with a much greater bandwidth than could be obtained from SCADA. For example, it has been revealed that in some instances, the power flow on a circuit shows an occasional burst of a relatively high-frequency component. Such a thing could be the result of a “hunting” control system, one that is marginally stable. Sometimes a mistuned power system stabilizer is evident in the voltage observed at some location. In fact, while the addition of a PMU to a system has always made possible the measurement results expected, there have been many times when unexpected results were also obtained. Adjustments to the power system to resolve such matters are bound to be beneficial.

The development of the technology has gone hand-in-hand with changes to the standards, and more and more aspects of the measurement are now covered by standard. For example, the PMU standard has always called for the measurement of the rate of change of frequency. However, it was not until the most recent version (2011) that there was any mention of verifying the accuracy of the result of the measurement.

Even now, there are some users of PMU data who are not satisfied with all that transpires in the measurement process. To some extent, that is the result of a failure to define the terms of the measurement. That failure, in turn, is the result of evolution in the applications of the results of the measurement. These are the aspects of the state of the art that led to the development described in the next section of this report.
2.0 Developments

A phasor measurement unit is a rather special measuring device, but it is essentially a measuring device. It should be understood as a measuring device. Before we can understand the device and whatever defects it has, it is worth examining what it means to make a measurement. That is a topic in epistemology, rather than technology, and one that is not widely understood. We begin by presenting some original material developed in the process of understanding what a PMU does. We continue by examining why it may not be satisfactory.

2.1 The Nature of Measurement

In general, measurement is a process that results in finding a value for a parameter in an expression such as this:

The value of the unknown was found to be P units.

The “unknown” is being described in some way. While the “description” is always some sort of shorthand, the P could be a multiplicity of parameters, and the term “units” could be any of our standard things, such as seconds or volts. And the expression could be written in the form of an equation. But the principle is the same: the result of the measurement is a value for a parameter, or a set of parameters, of this kind. In essence, some specific form of data compression is being done on a signal.

The reason for the measurement is not often questioned. Indeed, an informal survey of books on the topic of measurement done by the first author of this report has revealed nothing on the topic, in books for children or books for college. A children’s book on the topic could begin by saying that one’s mother needs to measure one’s waist so she can select a belt to hold up one’s trousers from the many for sale at the store, without taking one to the store. A college book could say that the current in a load is measured so the appropriate wire size for it can be selected. But these things do not seem to occur to authors, who evidently assume that everybody is born knowing why we make measurements.

The fact is that measurements are made to summarize or reduce the amount of information being handled. The need may be to reduce bandwidth in a communication system, or to reduce something to a convenient or understandable description. But the essence is always reduction in information.

The result of a measurement is something obtained at the end of what may be a long process. To understand the role of the measuring equipment in this process, consider Figure 1, an abstraction of the process of measurement. We assume a digital measurement instrument. The hardware and software of the measuring instrument are buried in a multitude of interconnected influences.

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5 In order to be clear, it is conventional to describe the end of the process as the result, rather than the measurement. The reading on an instrument (which may be the result of a measurement) is also known as its declared value.
6 A metrologist would not make a statement like the one above without qualifying it. A more complete statement, after multiple measurements have been made, is of the form

The value of the unknown was found with 95% probability to be within 1% of P units.
What this sort of qualification implies is that the measuring process is associated with some uncertainties and errors, and these uncertainties and errors can neither be known exactly nor reduced to zero.
The measurement sequence was anticipated very nicely by George Carey Foster more than a century ago:

Before methods of measurement can be devised, it is evident that clear conceptions must be formed of the things to be measured. Such conceptions usually grow up by degrees in many minds from indistinct beginnings, until, in some one mind, they take definite shape and receive the precise expression which makes it possible for them to become the subject of mathematical reasoning.

Carey Foster⁷ perceived that measurement is a process that starts with a concept. As we shall see, there are lots of places to disconnect the result of the measurement from this indistinct beginning of the process.

The concept may be, as Carey Foster said, indistinct. Before a measuring instrument can be made, something more definitive is needed. This is the description that metrologists know as the *measurand*, the thing that is to be measured.

However, a measurand is still a conceptual thing, and cannot itself be measured. It is a description, or rather a *definition*, of the thing to be measured, and so it is used to find something real to present to the measuring instrument. This is called the *realized quantity*.⁸ The realized quantity is presented as a stimulus to the measuring instrument, which executes some kind of data compression algorithm to produce the *result* of the measurement as its *declared value*.

---


⁸ It has to be said, however, that most metrologists would accept the use of the word *measurand* to mean *realized quantity* provided the meaning was clear from the context.
Note that the algorithm is designed with reference to the measurand, which plays a central role in the process. The measurand takes into account the needs of the application for the result, and it guides the selection of the realized quantity and the selection (or design) of the algorithm.

Some examples are given below to clarify the situation.

**Example 1.** Suppose it is desired to measure the voltage on a low voltage ac circuit, such as the 120-V output of a regular outlet. The outlet has presumably been selected as the realized quantity for some reason to do with the application – perhaps it is just to show that the voltage is within acceptable limits. What is needed as the result of the measurement is a single steady value (such as “120 V”), and there is no interest in the frequency of the voltage, or its distortion. An averaging of the rectified voltage would suffice as the data compression, on the assumption that the relation between this average and the accepted (but more difficult to implement) rms value can be assumed.

The measurand is therefore some definition of the voltage that expresses the voltage level as the required declared value. It may make a statement to the effect that the realized quantity must be sinusoidal in character, so that the relationship between average-rectified and rms can be assumed in the algorithm.

**Example 2.** Suppose the voltage at the same outlet is to be more completely characterized. The realized quantity is the same, but the measurand now must make a broader statement. It might, for example, say that values are required for the frequency and the total harmonic distortion. The implication now is that there is no requirement that the realized quantity be sinusoidal. In that case, the voltage level would probably have to be defined as the rms value, and the algorithm changed accordingly.

**Example 3.** Suppose the voltage at the same outlet is to be even more completely characterized. In addition to measuring the frequency, we ask for a measurement of the frequency of the harmonics in the signal. The solution is clear enough. But the need highlights the importance of the application in the measurement process. As the application changes, the measurand changes and the algorithm must change.

So far, we have implicitly made an assumption about the realized quantity. Let us now make that assumption explicit: we have assumed that the signal being used as the stimulus to the measuring instrument is stationary. Whether or not we realized it, the very notion of sinusoidal quantities and the term “frequency” require that the signal being observed is stationary. What that means is that the signal is the same for all time: the values that characterize the signal do not change depending on when the measurement is made. The equations used in the standard (IEEE Std C37.118.1-2011™) in defining the measurement to be made by a PMU are all stationary equations.

Yet the power system is not stationary.

When we remove the assumption of stationarity, we remove at the same time the definition of frequency. That statement might not be very appealing, but it is true. Recall how we use the phasor diagram. We draw the diagram on the assumption that the speed of rotation is constant (though of course, the speed on our drawing is zero). We write the voltage as

\[ v(t) = \sin(\omega t) \]  

(1)
and we say that \( \omega \) is the angular frequency, a constant in the diagram. Equation (1) has all the assumptions of stationarity.

The argument of the \( \sin \) term is an angle. The product \( \omega t \) can be called the phase angle (though that use is not universal, and is discussed below in Section 2.2), and we say that \( \omega \) is the rate at which the phase angle is changing.

If \( \omega \) is not constant, we imagine a situation in which the frequency is varying. In other words, in this method, we define frequency as the rate of change of angle, and we imagine (somewhat without rigor) that the value can change.

One useful way to bring about a situation in which \( \omega \) is not constant (and to reintroduce some rigor) is to modulate it. Consider the usual definition of a frequency-modulated wave:

\[
\omega(t) = \omega_c + \omega_m(t)
\]  

(2)

Here \( \omega_c \) is the carrier frequency and the time-varying \( \omega_m(t) \) is the modulation added to that constant frequency. The \( \omega(t) \) can be called the instantaneous frequency. If we integrate (2) to find the phase, we get

\[
\phi(t) = \omega_c t + \int_0^t \omega_m(t) dt
\]  

(3)

If we now further assume that the modulation is just a single frequency we find that the voltage is given by

\[
v(t) = \sin \left( \omega_c t + \frac{K_m}{\omega_m} \sin(\omega_m(t)) \right)
\]  

(4)

where the coefficient \( K_m/\omega_m \) is the modulation index. It is well known that expansion of this equation leads to an infinite number of sidebands. What does it mean to state a frequency for a signal that has sidebands going off to infinity? If we cannot explain what it means, how can we make a measurement of it?

The lack of a good definition of frequency, except as a stationary quantity, is explored in the next section.

### 2.2 Phase and Frequency

Engineers in general, and power engineers in particular, are familiar with the notion of “angular frequency.” We use the concept to imagine a generator rotating at some speed producing an output varying at some rate that we can represent in a phasor diagram. We struggle when the speed is not constant. For most power engineering problems, we ignore the possibility of operation at off-nominal frequency. But we require the PMU to measure the frequency, thereby showing that we are at some level aware of the possibility that the frequency is worth measuring, and not constant at all. We ask the PMU to go further and tell us the rate of change of frequency, and that gives more problems, for a number of different reasons.
First, the way the PMU measures frequency is by taking the differences between successive measurements of phase. The differencing operation is known to be sensitive to noise in the signal, and the impact of that noise is not something that has been studied. The PMU measures frequency change by taking the differences between successive measurements of frequency. The noise impact is compounded.

The method overlooks that deeper problem: We have not defined what we mean by frequency when the speed of the machine changes. The struggle we have when the speed is not constant is perhaps not well known, but it is not new. As long ago as 1922 John Carson of AT&T wrote a classic paper (it was so called when IEEE re-published it in 1963) in which he addressed the definition of frequency [3]. His topic was frequency modulation, at the time being proposed as a way to reduce bandwidth. (He showed that there is no bandwidth saving over amplitude modulation.)

Still on the topic of frequency modulation, a very readable discussion of the views of several workers in the field was given by van der Pol in 1946 [4]. The matter of defining phase, frequency and instantaneous frequency is examined. Historically, several workers have used the word phase to describe the entire argument of the cosine expression in quantities such as (5) below:

\[ \cos(\omega t + \phi) \]  

(We adopted that use in our discussion of (1) above because the argument was only the product \( \omega t \).) In his paper, van der Pol argues in favor of that definition, because

This definition has, among others, the advantage of enabling one to speak of the phase difference between two oscillations of different frequencies. This phase difference is then simply a linear function of the time, just as one phase by itself is already such a function of the time.

The difference between the definition favored by van der Pol (phase is the whole argument of the cosine function, namely \( (\omega t + \phi) \)) and the one in common use today (phase is just the \( \phi \) component) is that the present definition does not allow one to speak of the phase between two signals of different frequency.

That observation is important, because the PMU does precisely what it is not allowed to do! The PMU measures the difference in phase (based on the “modern” definition) between the reference signal at the nominal power frequency and the actual power system, which is always close to the nominal frequency, but rarely at precisely the same value. Since this is not a permitted operation, one is entitled to speculate on the meaning of the result. It may be – indeed it is likely – that the defects in any two such measurements cancel when they are subtracted in order to find the value of the phase between two physical locations. If so, that is convenient, but somewhat less than satisfactory.

The point is this: if the frequency is not constant, life just gets difficult! Boualem Bouashash⁹ put it more succinctly when he wrote

Nonstationary signals in particular do not lend themselves well to decomposition into sinusoidal components. For such signals, the notion of frequency loses its effectiveness, and one needs to use a parameter which accounts for the time-varying nature of the process.

---

To deal with the difficulties of measuring what is surely not a constant frequency in a PMU, the assumption is made that the signal is “nearly stationary.” The frequency is allowed to change only a little, and at a low rate. That is to say, only a narrow band of low amplitude low modulation frequencies is of interest, and that the modulation can be considered to occur at a single frequency at a time.\(^\text{10}\)

For the purposes of calibrating a PMU, a single frequency FM such as that is straightforward. To see the limitations imposed by that statement, it is necessary to understand what is meant by calibration.

### 2.3 Calibration

We noted earlier that any measurement involves uncertainties and errors that cannot be completely eliminated. Calibration, usually thought of as a way to “transfer accuracy,” is the process used to quantify the uncertainties and (by adjustment) reduce the errors. Calibration compares the declared value of an attribute of a calibrating artifact, such as a reference standard, with the declared value of an attribute of the unit under test. There are several ways to do this comparison that are different in principle. The choice depends on the nature of the unit under test; specifically whether the unit under test is a generator of a stimulus, or a measuring device. The choice also depends on whether or not a high-quality version of the stimulus is available.

#### 2.3.1 Unit Under Test is a Source

A simple configuration can be used if a property of the unit under test provides the stimulus. For example, the unit under test could be a signal generator, or a voltage source. The unit under test declared attribute value is its indicated output. The calibrating artifact in this case is the measuring standard. The calibrating artifact’s declared attribute value is displayed or otherwise shown, as in Figure 2.

![Figure 2 Calibrating a source](image)

\(^\text{10}\) If multiple frequencies are applied at one time, the spectrum is not simply the sum of the separate spectra, because the process is nonlinear. Instead, the relative amplitudes are affected by the individual values of the modulation index for each component of the modulation.
2.3.2 Unit Under Test is a Measuring Instrument

In a reversal of the previous configuration, the calibrating artifact is a standard source providing the stimulus. The calibrating artifact’s declared value is its indicated value. The unit under test is the sensor. The sensor responds to the stimulus and drives a display. The displayed reading is the declared attribute value, as in Figure 3.

![Figure 3 Calibrating a measuring device](image)

2.3.3 No High-Quality Stimulus Exists

In the third configuration, no “high-quality” stimulus is available. Instead, a low-quality stimulus is supplied by a source external to both the calibrating artifact and the unit under test. Each artifact responds to the stimulus and drives a display. The displayed readings are the declared attribute values of the calibrating and calibrated unit, as in Figure 4. This process is used, for example, when the source is not particularly stable: the unit under test and the measuring standard should agree, even if the stimulus value changes. (Current transformer calibration using a current comparator is in this category. That way, the large current needed may not have to be particularly well controlled.)

![Figure 4 Calibration using (relatively) low-quality source](image)
The procedure of calibration, as shown here in simplified form, is part of a sequence that may contain many repetitions, from the national metrology laboratory down to the field calibration. For a critical measurement, it may be that the acceptable tolerance is “tighter” than it would be for a less demanding application. The essence of the process is the same nevertheless: declared values are compared. The difference is an indication of error or uncertainty in the lower-level device.

The entire process of calibration relies on the requirement that the calibrating standard must produce (or measure) exactly the same thing as the unit under test. It would make no sense – obviously – to use a thermometer to calibrate a voltage source. It is less obvious whether a laboratory signal generator (or an arbitrary function generator) makes sense as a calibration source for a PMU.

2.3.4 Calibration and PMUs

Certainly, a signal source such as an arbitrary function generator might allow one PMU to be compared to another, but it is not possible to attach meaning to the differences found in terms of the original concept behind a PMU. The problem arises because the function generator with frequency modulation is not very representative of the sort of signal that is going to be “seen” by the PMU in the real world. Similar situations arise in other fields.

For example, a high-quality radio receiver may have excellent sensitivity, and it may be able to receive signals as low in amplitude as the atmospheric noise at any frequency. But in the real world, the small signals it receives may be close in frequency to very strong signals. To know whether the receiver will work under such circumstances it is not sufficient just to know its sensitivity. A realistic calibration of the receiver takes that into account.

Figure 5 shows the effect on the observed “frequency” of the loss of a generator on the WECC system.

![Figure 5 Generator loss effect on system frequency](image)
It can be seen in Figure 5 that the system frequency drops very rapidly in the moments following the loss of a
generator, and recovers more slowly as the various control systems begin their corrective actions. It is very
obvious that “frequency” in the power system is not a stationary quantity.

It is pointed out that the authors of this present report, while they are critical of some aspects of the metrology
of the PMU, regard results such as Figure 5 as evidence that the devices are actually working well and are
furnishing useful results. The multiple colors of the graph correspond to the measurements being made by
several PMUs, and it is clear that they are in good agreement.

However, it is also clear that the change in frequency that takes place when the generator is tripped is abrupt.
For all practical purposes it seems to be a step in frequency, and is not at all like a gently modulated FM. How
is such a thing to be represented in a calibration procedure?

There is also evidence that suggests noise in the signal is affecting the output. At about 03:39.40 at least one
of the PMUs gives a result that indicates the frequency has returned to its nominal value. That sudden return
is not physically possible.

An arbitrary function generator could, in principle, be programmed to produce an output that was
representative of the power system that a PMU is designed to measure. This is a case where principle and
practice do not converge. To make a function generator produce a signal that will result in a PMU giving an
output that is like the curve in Figure 5 assumes that the result in the figure is exact. That is not the nature of
calibration: calibration goes beyond merely making all devices alike: it is aimed at ensuring their accuracy
with respect to a standard unit.

Thus, the question must be asked and answered: What should be the output of a frequency-measuring
instrument when a large generator is dropped from the power system? It seems no-one knows.

The power system is not so well described that the necessary parameters can simply be fed into a function
generator, so that the measurement device should then yield estimates of them as its output. Another way of
expressing the problem is to say that the independent measuring standard for the method does not presently
exist. The PMU standards, for example at NIST, are based on the same faulty phasor definitions as the units
that might come under test there.

To illustrate the problems in calibration, we include as
Figure 6 the results of some calibration work done at NIST.11 The frequency here is alternately being ramped
up, held constant, ramped down and held constant. The declared value is not given for the “corners” where the
system transitions from one mode to another, because the situation is undefined. In other words, it is not
known what the result should be. In the words of the authors, “The calculated values for periods that span any
transitions do not have correct values to use as a reference.”

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Unfortunately, that nicety is not allowed the PMU, which must and will continue to report values even if the input signal is not a sine-modulated wave. While it is perfectly possible to look at the gaps in Figure 6 and judge that the missing information could easily be filled in by extending the straight lines until they touched, remember that the PMU does not have that luxury: it must furnish data in real time. It cannot go back and fill in missing results.

In this sort of test, the PMU will be presented with a waveform such as the one shown in Figure 7, which shows a period of about two cycles. The curve of the realized quantity to the left of the zero of time corresponds to a signal frequency of 59.5 Hz, and to the right the frequency is exactly 60 Hz. The amplitudes are the same, so there is no discontinuity at zero time, even though there is a step in frequency. But the signal enters the measurement window slightly after its peak value.

It seems that the PMU will deal with this waveform. Why not: the waveforms of signals from the real power system are often much less orderly than this.

But it seems we have no way to know whether the declared value is correct, so we leave a gap in the record.

It seems to us not acceptable to ask the PMU to measure things that we cannot calibrate. It seems deeply unsatisfactory that for a waveform that “looks” well-behaved, we have no way to know if the result of a measurement is acceptable.

If we cannot calibrate the system with this waveform, how can we have confidence when the waveform is even less orderly?
2.4 A Perspective on Digital Measurements

Several years ago, a company called Yokogawa made a new kind of wattmeter. It implemented the measurement digitally, operating on a stream of sampled data from the system being measured. By careful design and implementation, the quoted uncertainties in the measurement were significantly lower than those in the standards used in various national metrology laboratories. That wattmeter can be regarded as having started a “digital revolution” in measurements. For all practical purposes, the measuring instrument of old has been replaced by the digital instrument. The instrument maker that carefully crafted the delicately balanced mechanisms of Kelvin and Maxwell’s day has been replaced by a software engineer. If you can define something, the new “instrument maker” can measure it.

But the definition has to relate the measurement to the real world. An example of what can happen otherwise was reported quite recently [5]. The electric meter of a large consumer of BC Hydro was replaced. Before the replacement, the customer load power factor was 0.95. After the replacement, it was 0.88, as a consequence of which a penalty was automatically added to the electric bill. Since the load had not changed, the customer asked for an explanation.

What had happened was this. The two meters (both digital) had the same realized quantity, but they had different definitions for the measurand, and hence different algorithms. Both were correct on the assumption of sinusoidal values, and both had been certified correct by Measurement Canada. However, the harmonic content of the load resulted in the two calculations not giving the same result.

One meter measured $Q$ by evaluating each harmonic term separately and summing them. That is, it calculated $Q = \sum V_h I_h \sin \theta_h$, where the subscript $h$ is the harmonic number. It then found the apparent power from $S = \sqrt{P^2 + Q^2}$. The other meter calculated $S = VI$, that is, the rms quantities were directly multiplied. In both cases the power factor was then found as the ratio of real to apparent power, $PF = P/S$.

The root of the problem is the definition of reactive power. What reactive power? As is happens, there is more than one definition. A study for NEMA (the National Electrical Manufacturers Association) found at least ten definitions being used in VAr-hour measurements [6]. While the various methods give the same solution if the waveforms are sinusoidal, the results differ if the waveforms are distorted.

What the digital wattmeter demonstrated is that if one can define a thing well, one can measure it digitally with almost arbitrary accuracy. One can sample the signal with extreme precision, and one can write an algorithm to implement the definition.

In contrast, what the energy meter demonstrated is that if one does not define a thing well, one can obtain all the decimal places one wants, and yet the result of the measurement will not have much meaning.

We are in that unhappy situation with PMUs. We have a definition for the measurand that is plainly wrong, because it is stationary. Any algorithm based on a faulty measurand is likely only to add imperfections.

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12 Measurement Canada is responsible in Canada for ensuring the accuracy of measurements. They evaluate and certify measurement instruments.
2.5 New PMU Method

The foregoing discussion hints that we would benefit from abandoning methods based on taking the derivatives of things such as measured phase, and examine directly the signal we are trying to characterize. We should accept what Boashash taught (that one needs to use a parameter which accounts for the time-varying nature of the process), and we should abandon the restriction that the measurand is a stationary sinusoidal quantity.

2.5.1 Proposed Solution

If we allow that the three actual parameters of interest of our realized quantity (amplitude, frequency and phase) are not stationary, the next simplest assumption is that they are changing at a constant rate.\(^\text{13}\) In that case, the equation that describes the waveform of the signal should have six parameters, not three. By analogy with the equation of the phasor, one could change the phasor version:

\[ x(t) = (X) \cos((\omega)t + (\varphi)) \]  

into this:

\[ x(t) = (X + C_X t) \cos((\omega + C_\omega t)t + (\varphi + C_\varphi t)) \]  

In this equation, each of the coefficients is modified from the phasor version to include an explicit variation with \( t \). The amplitude multiplier becomes \( X + C_X t \), the “frequency” is \( \omega + C_\omega t \), and the “phase” is \( \varphi + C_\varphi t \). The constants \( C \) are the factors for the rate of change, applied as indicated by the subscripts. If the constant is zero, the change vanishes, otherwise the change is linear with \( t \), as required for a constant rate of change.

This equation has broadly the same form as the usual equation for the phasor, except that the three parameters of relevance to the phasor are each replaced by two terms, the value and its change, where the change is calculated as the product of time and a constant rate of change. Note that the rate of change need only be constant during the interval of interest. It is not constant for all time.

The method does not merely give the point-value of the quantity being measured, or the average value. It gives enough information to calculate the value throughout the short interval as it changes.

Equation (7) must be modified slightly from the form above, however. The term \( C_\varphi t \) cannot be distinguished from \( \omega \) and is put with like terms \( \omega \) and \( C_\omega t \), leaving \( \varphi \) without an explicit rate of change.

\[ x(t) = (X + C_X t) \cos((\omega + C_\omega t)t + (\varphi + C_\varphi t)) \]  

\[ = (X + C_X t) \cos((\omega + C_\varphi + C_\omega t)t + \varphi) \]  

\(^{13}\) This is one of those things that seems so obvious in retrospect.
The equation is not the equation of a phasor, and the solution applies for only the time considered by the length of the measurement window. It is, nevertheless, of the same general form as the conventional equation for a phasor, and therefore its solution is likely to be useful.

### 2.5.2 New Definitions for the Measurand

Since equation (8) does not describe a phasor, it is proposed that the names for the terms on the right side be changed by the addition of the word *apparent*, so that we find the *apparent amplitude*, *apparent frequency* and *apparent phase*. This use of the word apparent is essential, as it implies a revised definition for the frequency and the phase. The modification is in keeping with the use of the word in IEC-60270, where the partial discharge is characterized by something that standard calls *apparent charge*. Apparent charge is not what causes the discharge, it is the declared value of a measurement. In the same way, apparent frequency would be the declared value of a measurement, and not something that necessarily has any reality beyond that statement.

It is also suggested that the term *instantaneous frequency* be reserved for the case that the rate of change of frequency is non-zero. The term $\omega + C_\varphi$ is thus the apparent frequency and $\omega + C_\varphi + C_\omega t$ is the instantaneous frequency. The word *frequency* (with no modifier) can thus be reserved for a phasor quantity, ie, one that is stationary.

If these definitions are adopted (and assuming that the equation has a solution that can be found), the effect on the power system when a large generator is dropped might best be described by a number of terms. If the conditions really were constant for a while prior to the loss of the generator, one might say that at the start of the event the frequency had a value of such-and-such. For the duration of the measurement window that followed, one might say that the apparent frequency was so-and-so. At the start of the window the instantaneous frequency was this, and at the end it was that.

The rate of change of frequency value is really the rate of change of instantaneous frequency. The frequency $\omega$ does not change (by definition) and the apparent frequency $\omega + C_\varphi$ does not change across the width of a measurement window. Only $C_\omega t$ changes during the window.

It is fair to ask what equation (8) tells us about the phase. Because of the practical matter of solving the equation, we combined the rate-of-change-of-phase term with the frequency. They are dimensionally the same, and we could see no mechanism by which they could be separated in a measurement. That had the effect of “freezing” the value of phase. Although the frequency might be changing, the phase (using the modern interpretation of the word) was the same throughout the measurement window. Yet in some way we cannot but sense that is not “right” because we began by assigning a value $C_\varphi$ to the parameter.

The problem is that what “looks like” phase is something that changes during a measurement window even if the frequency is constant. That is because of the problem pointed out by van der Pol, that our usual (modern?) definition of phase does not allow one to talk about the phase angle between two signals of different frequency.
We consider that situation in Figure 8, which shows a cosine wave that has a constant off-nominal frequency \( f = 59.5 \) Hz. The wave started at \( t = 0 \) with no phase shift. Twelve nominal-frequency cycles later (at time 0.2 s) there is an obvious phase shift. What looks like a phase shift is gradually accumulated because of the difference in frequency between the reference frequency and the signal at 59.5 Hz. One nominal-cycle later, the phase shift is slightly different, and so on into the future.

The reason for using the word “apparent” with respect to phase is now clear. The “obvious” value is not constant, as the frequency is not the same as the reference frequency. Less obviously, “phase” is not mathematically defined for two signals of different frequency using the conventional definition of phase (ie, not the one favored by van der Pol).

### 2.5.3 New Definitions for the Time

However, it is quite evident from Figure 8 that it should be possible to express something phase-like about the relationship between the realized quantity and the reference. An analogy can be used. The statement that phase is not defined when frequencies are different is true, but the implication that it is therefore not possible to measure the angle between the hour hand and the second hand of a clock is not true. A snapshot will allow a measurement at some particular time. In exactly this way, Figure 8 allows a snapshot of the phase at 0.2 seconds. It is this quantity that we are calling apparent phase.

We need to justify selecting a value for the time of our snapshot. Fortunately, we have a justifiable selection already defined for us.

Because the PMU standard defines the times for which measurements must be reported, we can use these times to take our snapshot, as in Figure 8. The time 0.2 s shown in the figure is one of the valid times for reporting measurements at 60 frames per second (fps) or 10 fps, for example.

To extend the analogy of the clock, we can take the photograph at any time we want. We could choose to take our photograph every time the second hand was pointing to the “3.” The result could then be defined as always having zero degrees for the angle of the second hand. The difference between the hour hand and the second hand would therefore always be simply given by the angle of the hour hand.
By analogy, we choose to calculate the power system angle at the reporting times given by the PMU standard. At these times the reference wave is always the same as it is at time $t = 0$: zero degrees. The angle of the signal evaluated at that time is therefore the angle with respect to the reference, with no further arithmetic required.

### 2.5.4 Curve Fit

The most obvious way to find the values for the five parameters that are needed to fully define the curves of Figure 8 is to do a curve fit, with a “local time” that is zero at the center of the interval being analyzed. For the calculation of the signal in Figure 8, the local time would be set to zero at 0.2 s of real time. In imitation of the type P PMU, we chose as a demonstration to fit a section of data two nominal-cycles in duration, shown as the heavier of the two solid lines in the figure.

We fit the parameters of an equation like the one above, whose form is constrained, to the signal we are measuring. (It may be that other parameters could be included: for example, a dc offset could be taken care of by changing the form of the equation. It should also be possible to account for harmonics.) The output of the fitting process is the five parameters, and a further number indicating the quality of the fit.

The limitation to the process is given by the amount of information in the signal and the amount of noise. At the moment, we have fitted synthetic data, and not real-world point-on-wave data that might include noise.

It should be pointed out that the assumption of constant rates of change is likely a fair approximation if the period of interest is short. That is itself a matter for investigation. The power system very likely allows that assumption to be valid for the short time windows of interest for protection (when typically two cycles are used for the window) and perhaps also for metering, where more cycles are included in the window.

### 2.5.5 Tests of the Method

We have so far tested the method using data created by a spreadsheet. (This way we know the exact values that should be found.) One set of results is shown in Figure 9, from a MATLAB curve-fit program.

In these graphs, the horizontal axis is time. The number shown is the number of the pairs of cycles analyzed: there is a two-second period. The plots were obtained by “joining the dots,” not the best method to show the apparent phase data (because the instant of wrapping is shown instead as an interval), but it is the default.

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14 The idea of using a curve-fit method is consistent with the advice offered by Arun Phadke (another of the originators of the first PMU) who commented that whatever method was used to find the frequency, it should use all the data, and not rely just on zero-crossings.

15 During tests, we have been able to obtain an accurate value for the apparent frequency by submitting approximately half a cycle of the input signal. Since the method is not using zero-crossings, the half-cycle may include only one zero crossing. We have not yet explored what might be the smallest amount of data to yield a useable value. It is reasonable to expect that the signal-to-noise ratio of the input will be a factor in this result, and if the signal is “clean” rather few sample points may be needed.
The fit seems to be giving exact results. The apparent frequency is seen to ramp from 50 to 52 Hz during the test, and the phase increases as a parabola, as would be expected. The test is unrealistic in that the sample rate is a few times higher than may be expected in a PMU.

In our trials, we also obtain a number indicating the quality of the fit. While we have so far made no use of this number, it may turn out to be valuable in the real world, indicating something about the confidence to be placed in the result of the measurement. In other words, it may be possible to relate this number to the statement of the measurement result as it would be made by a metrologist, as in footnote 6, page 6.

For implementation purposes, it is worth noting that the result of a measurement of a short section of signal can be used to estimate the values for the next segment. That may be a consideration for speeding up the calculations in a PMU.
Figure 9  Synthetic frequency ramp
3.0 Future Work

After a long gestation period, we have devised a new method of using sampled data obtained from a power system to obtain results that may well represent the power system better than the results of a PMU measurement. The work of fully understanding what we have created has barely begun, however.

One of the things we would like to do (obviously) is to make tests of the method under a variety of conditions. Under the “clean” conditions we have so far used for test, and with the MATLAB solver, the method seems to run quite fast, and usually produces the expected results. We have not yet been able to explore the effect of having more or fewer samples in the data stream, nor do we know whether the method is fast enough for use as a PMU replacement. It may be that a purpose-built solver would be better.

Some of these anticipated test conditions should be realistic: they could be thought of as allowing the method to be classified at a high level of technology readiness if they show good performance in the noisy environment of the electric power system.

3.1 Noise

Noise in a power system comes from a variety of sources. There are many sources each capable of producing a little noise, and some sources capable of liberating considerable amounts of energy into the electromagnetic spectrum. The first kind, sources such as rain-induced corona on a power line, produce what is generally called Gaussian noise. Since the noise sources are not uncorrelated (they vary with the local electric field), the noise they produce is not truly white noise, but it does have a continuous presence and a wide spectrum. The second kind, sources such switching impulses or lightning, occur less frequently, but are capable of injecting larger amounts of energy right across the spectrum.

We have already commented that the state of the art of mission assurance for electronics for the power system was not very advanced back in the 1960s when transistors started to fail. Since then, transistor reliability has improved, and test standards have been developed. As an example of the latter, IEEE Std 1613 (IEEE Standard Environmental and Testing Requirements for Communications Networking Devices in Electric Power Substations), specifies testing communication networks in substations with an impulse voltage of 5 kV and a risetime of 1.2 µs.

The authors are not aware of any particular tests for functioning during a test. IEEE Std 1613, for example, states that the acceptance criteria for the dielectric test are that “no breakdown or flashover shall occur and no components shall be damaged. After the test the device shall still comply with all relevant performance requirements.” That rather limited view of mission assurance seems too lax for PMUs, which will continue to make reports of their measurements even when there is noise in the system. That noise may be corrupting the data, and might be the cause of “spike” such as the one seen in Figure 5.

It would be valuable to investigate the effect of noise in the PMU process, and to investigate the response of the new method compared to the previous ones.
3.2 Filtering

All digital measuring systems should begin with what is called an anti-aliasing filter. The purpose of the filter is to remove energy from that part of the spectrum above the Nyquist value, so that it cannot “fold” into the spectrum of interest and appear as a signal of interest. The filter must be analog, because the signal has not been digitized at this point in the signal chain.\(^\text{16}\)

After that, the filtering in the measuring instrument is in the digital domain. The variety and capability of the filtering is quite impressive. Filters have a large influence on the processing involved in PMU measurement. A glance at the standard for PMUs reveals that much of the filtering is specified by standard. Compensation for group delay in filters is therefore part of the processing, and testing that compensation is part of the required testing. What that means is that while the measurement window may be centered on the reporting time (such as the time 0.2 s in Figure 8) filtering delays could have the effect of making the result of the calculation actually applicable at a slightly different time.

At this point in the development of the new method, we have not incorporated any filtering. It will be recognized that the action of the curve-fit is itself a kind of filtering. Certainly, the present work has used a sharp cutoff in the time domain. It may be that that represents sufficient filtering for a PMU-like output. It would be worth investigating, as the method so far seems to work even for very short measurement windows. If the windows do not overlap, it may be possible to regard each report as an independent measurement (see also footnote 16, page 20).

3.3 Curve-fit Options

We have tested the method so far only by writing MATLAB scripts to implement the curve-fit. From time to time we have seen problems with the fit. However, the general-purpose curve-fit used by MATLAB may not be well-suited to this particular problem. We have given some thought to how a curve-fit might be developed.

Rather than draw a distorted sine-wave to illustrate the curve-fit method, it seems it might be instructive to draw a uniform sine-wave and distort the space it is in. This concept is illustrated in Figure 10, where a box is used to represent the space that the sine-wave is in. The left part of each of the drawings in the figure is a reference, and on the right side it is modified as indicated. In each part of the drawing, the vertical dimension of the box affects the perceived magnitude of the signal, and the horizontal dimension reflects the perceived time. Since the wave is constant, the change in the box may seem counterintuitive. For example, Figure 10 (b) represents a situation where the signal is growing larger as the time increases. Figure 10 (d) shows a signal whose frequency is decreasing.

\(^\text{16}\) It may be noted that some PMUs do not use an anti-alias filter at the front-end, on the supposed grounds that there is no energy in that part of the spectrum, and therefore nothing would be gained. That seems to be in the nature of an assertion, rather than a universal truth, and it surely leaves the system vulnerable to noise. One may speculate that the decision to omit an anti-aliasing filter has its origins in confusion between the reporting rate of the PMU (which could be as high as the power frequency itself according to Table 1 of the standard) and the sampling rate (which will be many times higher than that). Since the measurement windows of adjacent reports can overlap, however, it should be clear that the measurements are not independent uncorrelated measurements, and therefore a second application of the sampling theorem is justified. It is this filtering, related to the reporting rate and not the sampling rate, that is specified in the standard.
There are no other ways to distort the box that make sense in interpreting the signal as a phasor-like quantity. The five changed boxes in Figure 10 seem to be a complete set in that sense. The only additional changes that might be useful are to raise or lower the box, a change corresponding to a dc offset. A changing offset would be represented by making the box a parallelogram. These ideas are shown in Figure 11. (At this time, we have not attempted to fit data that include an offset.)

The use of the box as in these figures suggests that the fit method of some future curve-fit algorithm might benefit from examining the distribution of the residuals in this space.

Residuals are the difference between the observed value of the data being fitted, and the calculated value. A general purpose curve-fit routine works by minimizing the sum of the squares of the residuals. For our situation, that may not be the best method.\footnote{We recall the work of Brian Stott. Brian showed that by solving the power flow problem in two separate parts (the real and imaginary parts) and then combining results, the total solution time for the iterative solution of the power flow was greatly reduced. His method was called the \textit{fast decoupled} load flow, and it has been the basis of the load flow solution ever since he made the case. We speculate that since the form of our curve-fit equation is fixed, a less general-purpose curve-fit than the one we have so far used would be significantly better, or faster, or both.}
In the case of the curve fit to our equation, it may very well be that residuals should be thought of in both the $x$ and the $y$ direction, rather than just the $y$ direction as is conventional. Relating the residual to the estimated value of the equation may also be a way to improve the speed or the accuracy of the fit. For example, normalizing the residual based on the observed value would reduce the importance of the values near the peak of the cosine wave, and emphasize the values nearer to the $x$-axis when it came to evaluating the amplitude coefficients (see Figure 10 (a) and (b)). Whether that would actually help the fit has to be studied. Similar changes in the customary curve fit, regarding the frequency and phase parameters, are surely worth investigating.
4.0 Summary and Conclusions

Phasor measurements are without doubt extraordinarily valuable and informative. The number of phasor measurement units in service continues to increase, to the great benefit of the power system. And yet the measurement method used is deeply flawed.

From time to time, it is evident that there are inconsistencies between measurements, both during test and during use. Because the quantity being measured is not well defined, it is not possible to say what is right or wrong when two systems give divergent results. Only if there is a solidly supportable definition for what is being measured can a metrologist be sure that it is, indeed, being measured.

Existing PMU technology requires that the result of the measurement of the input signal be expressed as if it were a phasor. The equations in the IEEE standard are all the equations of a phasor. And yet we have seen that the behavior of the real power system is not described by phasors. Indeed, the rate of change of frequency of a phasor is defined as zero, so on its face that would not seem to be worth measuring.

We have shown that, instead of imposing the requirement that the input signal to the measurement be described by a phasor, the assumption of stationarity can be relaxed. The result of changing the definition of the input signal is that a new measurement technique is possible. The new method furnishes values for some new (and concretely defined) parameters. “Apparent frequency” is a parameter that is constant across a measurement window. “Instantaneous frequency” can change during a measurement. The latter parameter is associated with a well-defined rate of change. No differentiation or finite-difference calculations are used in the definitions.

Preliminary tests of the new measurement method are given. They seem promising. Further work on the method is justified.
Appendix A. Testing with an RTDS File

We said above that an arbitrary function generator could, in principle, be programmed to produce an output that was representative of the power system that a PMU is designed to measure. The system setup that might accomplish that is shown in Figure 12. The arbitrary function generator produces the two waveforms at the left of the figure, and this signal modulates a voltage controlled oscillator.

![Figure 12](image)

Figure 12 PMU testing based on generating an arbitrary function

Here the function generator is used to produce a signal that is representative of the way a real power system behaves. For that to be possible requires that the power system is so well understood that it can be described so that the necessary parameters can be fed into the function generator.

The three waveforms shown in the figure have been adapted from PMU measurements of an actual generator-loss on a power system. While these could be digitized and fed onto a function generator, that would represent a sort of circular logic, because they were obtained from PMU observations. And in any case, while the function generator would be programmed, it would not be programmed with values of any of the parameters other than the one shown, likely the result of a frequency measurement made on a bus voltage. The value for ROCOF would not be a programmed-in value. The test would not allow the PMU estimate of ROCOF to be calibrated, therefore.

We also mentioned in our discussion of calibration that the power system might be modelled reasonably exactly by an RTDS. In this case the output of the RTDS would represent the “low-quality” stimulus of Figure 4. The system for this arrangement is shown in Figure 13.

![Figure 13](image)

Figure 13 PMU testing based on an RTDS

The problem here is that the information we get from the PMU still cannot be compared with the values we expect to get. The reason is simple: we still have no way of saying what the expected parameters are.

However, that does not mean that such a test is without value. The RTDS can be programmed to furnish waveforms that are very representative of the signals from a power system, and while that does not achieve a calibration of the PMU, it does give an indication of how it would perform in a real-world situation. And that is surely useful. It may even be possible to incorporate a noise test in such a setup.
References


From the footnotes: